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LETTER TO THE EDITOR

Stimulated bremsstrahlung in the presence of an intense electromagnetic wave

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Abstract. The influence of the strong wave $\boldsymbol{E}_0 \sin \omega_0 t$ on the stimulated bremsstrahlung at some frequency ω is considered. The asymptotics $E_0 \to \infty$ is investigated in two cases: when the electron velocity $\boldsymbol{v}$ is parallel to $\boldsymbol{E}_0$ and then the electron distribution is isotropic. The absorption coefficient $\alpha(\omega)$ is found to be proportional to E_0^{-1}. Resonances in the stimulated bremsstrahlung, which occur near the points $\omega \approx n\omega_0$, $n = 1, 2, \ldots$, are investigated. The conditions for negative absorption are discussed.

The nonlinear absorption of an intense electromagnetic (EM) wave due to the multiphoton inverse bremsstrahlung has been widely investigated both classically and quantum mechanically (Silin 1964, Rand 1964, Bunkin and Fedorov 1965, Pert 1972, Bunkin *et al* 1972). According to these (and many subsequent) papers the absorption coefficient α_0 of a strong EM wave may be obtained qualitatively if we replace the electron velocity v in the classical expression for α_0 (Spitzer 1964) by the electron oscillation velocity $v_E = eE_0/m\omega_0$ where E_0 and ω_0 are the field strength and the frequency of intense EM wave. Hence in the case $v_E \gg v$, α_0 decreases as E_0^{-3} with an increase of the field strength E_0.

We shall investigate the influence of an intense EM wave $\boldsymbol{E}^{(0)}(t) = \boldsymbol{E}_0 \sin \omega_0 t$ on the stimulated one-photon bremsstrahlung at some frequency ω. The corresponding classical calculations have been carried out earlier (Fedorov 1971) using the plasma kinetic equation method (Silin 1964). Here we shall give a quantum mechanical solution of the problem.

We consider an electron scattered by the Coulomb potential $V(r) = Ze^2/r$ in the presence of two EM waves $\boldsymbol{E}^{(0)}(t)$ and $\boldsymbol{E}(t) = \boldsymbol{E} \sin \omega t$, one of which ($\boldsymbol{E}(t)$) will be considered as a small perturbation. The one-photon emission (σ_e) and absorption (σ_a) cross sections may be derived from the first-order Schrödinger equation in $\boldsymbol{E}(t)$ and in the first Born approximation of the scattering potential V. The strong field $E^{(0)}$ should be taken into account exactly in the zero-order wavefunctions of Bunkin and Fedorov (1965). The result of these calculations is given by the sum over the number n of absorbed or emitted photons $\hbar\omega_0$:

$$\sigma_{e,a} = \frac{Z^2 e^6 E^2}{\hbar^2 \omega^4 m^2 v} \sum_n \int d\boldsymbol{k} \frac{(\boldsymbol{k}\boldsymbol{E})^2}{k^4} J_n^2\left(\frac{e\boldsymbol{E}_0\boldsymbol{k}}{m\omega_0^2}\right) \delta\left(\boldsymbol{v}\boldsymbol{k} - n\omega_0 + \omega \pm \frac{\hbar k^2}{2m}\right) \qquad (1)$$

where $J_n(x)$ is the Bessel function, $n = 0, \pm 1, \pm 2, \ldots$.

The total emission cross section $\sigma_T = \sigma_e - \sigma_a$ is simply related to the weak-field absorption coefficient, $\alpha = -8\pi(v/c)n_i n_e(\hbar\omega\sigma_T/E^2)$ where n_i and n_e are the ion and electron number densities (if $\sigma_T > 0$ negative absorption is possible with amplification coefficient $\tilde{\alpha} = -\alpha$).

Further calculations will be carried out for the most interesting case $v_E \gg v$ with the help of the large-argument expansion of the Bessel functions $J_n^2(x)$ averaged over the fast oscillations:

$$\sigma_{e,a} = \frac{Z^2e^5\omega_0^2}{\pi\hbar^2\omega^4 mv} \sum_n \int \frac{\mathrm{d}\boldsymbol{k}}{k^4} \frac{(\boldsymbol{kE})^2}{|\boldsymbol{k}\boldsymbol{E}_0|} \delta\left(\boldsymbol{vk} - n\omega_0 + \omega \pm \frac{\hbar k^2}{2m}\right). \tag{2}$$

This representation is valid everywhere except in a small range $e|\boldsymbol{E}_0\boldsymbol{k}|/m\omega_0^2 \lesssim \nu \equiv \max(1, |n|)$ which can be taken into account by the cut-off in the integral over $\boldsymbol{k}$.

At first we let the electron velocity $\boldsymbol{v}$ be parallel to the field strength $\boldsymbol{E}_0$. The integration in equation (2) over the solid angle in $\boldsymbol{k}$-space is readily carried out to give

$$\sigma_{e,a} = \frac{Z^2e^5E^2\omega_0^2}{\hbar^2\omega^4 mv^2E_0} \sum_n \int \frac{\mathrm{d}k}{k^2} \frac{\sin^2\theta + (3\cos^2\theta - 1)x^2(k)}{|x(k)|},$$

$$x(k) = \frac{n\omega_0 - \omega}{vk} \mp \frac{\hbar k}{2mv}, \qquad 1 \geqslant |x(k)| \geqslant \frac{m\omega_0^2\nu}{eE_0k} \tag{3}$$

where θ is the angle between the field strengths E_0 and E. The second inequality for $x(k)$ in equation (3) is connected with the cut-off procedure mentioned above. According to the large-argument expansion already used we should retain in $\sigma_{e,a}$ only those terms which are proportional to the large logarithm of the form $\ln(v_E/v)$. In this approximation equation (3) yields

$$\sigma_T = \frac{Z^2e^5\omega_0^2E^2}{\hbar^2\omega^4 mvE_0} \sin^2\theta \sum_{n=-\infty}^{+\infty} \frac{1}{n\omega_0 - \omega} \ln\left(\frac{2eE_0|n\omega_0 - \omega|}{mv\omega_0^2\nu}\right). \tag{4}$$

The most interesting feature of this result is the resonance dependence of the cross section σ_T from the emitted photon's frequency ω near the points $\omega = n\omega_0$, $n = 1, 2, \ldots$. In the neighbourhood of resonances the energy of an intense EM wave can be transformed most effectively into the energy of a weak wave or vice versa. The negative absorption is possible if $(n - \frac{1}{2})\omega_0 < \omega < n\omega_0$. The maximum value of the weak-wave amplification coefficient which can be achieved in the vicinity of resonances is readily estimated if we remember that the argument of the logarithm in equation (4) should be large, $|n\omega_0 - \omega| \gtrsim mv\omega_0^2 n/eE_0$. Substituting this minimum deviation from resonance in equation (4) and replacing the logarithm by unity we obtain $\tilde{\alpha}_{\max} \approx 8\pi Z^2e^6 n_i n_e/n\hbar\omega^3 m^2 vc$ which differs by the large factor of the order of $mv^2/n\hbar\omega \gg 1$ from the usual stimulated bremsstrahlung amplification coefficient (Marcuse 1962).

The quantum mechanical origin of the results obtained should be emphasized (the weak-wave absorption coefficient α is proportional to $\hbar^{-1}$). A more general investigation which will be published elsewhere shows that these peculiarities are specific to the case $\boldsymbol{v} \| \boldsymbol{E}_0$ we consider and disappear if the angle χ between the vectors $\boldsymbol{v}$ and $\boldsymbol{E}_0$, or the angular divergence of the electron beam, exceeds a small value of the order of $\sqrt{\xi} \equiv (2\hbar\omega_0/mv^2)^{1/2} \ll 1$. This deduction will be illustrated now by considering the case of a totally isotropic electron distribution.

The average values of the cross sections $\sigma_{e,a}$ (over directions of the electron velocity v) according to equation (2) in the case $v_E \gg v$ are given by

$$\bar{\sigma}_{e,a} = \frac{Z^2 e^5 \omega_0^2}{\pi \hbar^2 \omega^4 m v^2} \sum_n \int \frac{dk}{k^3} \frac{(kE)^2}{|kE_0|},$$

$$\frac{2m|n\omega_0 - \omega|}{\hbar k_0} \leqslant k \leqslant k_0 \equiv \frac{mv}{\hbar}\left[1 + \left(1 \mp \frac{2\hbar(n\omega_0 - \omega)}{mv^2}\right)^{1/2}\right] \frac{e|E_0 k|}{m\omega_0^2} \gtrsim \nu. \tag{5}$$

Carrying out the integrations and retaining only the logarithmically large terms we find

$$\bar{\sigma}_T \approx \frac{Z^2 e^5 \omega_0 E^2 \sin^2\theta}{\hbar^2 \omega^4 m v E_0} \ln\left(\frac{v_E}{v}\right)\left(\sum_{n=1}^{[(1/\xi)+s]} \frac{1-\sqrt{1-\xi(n-s)}}{n-s} - \sum_{n=1}^{[(1/\xi)+1-s]} \frac{1-\sqrt{1-\xi(n-1+s)}}{n-1+s}\right.$$
$$\left. + \sum_{[(1/\xi)+1+s]}^{\infty} \frac{1}{n-s} - \sum_{[(1/\xi)+2-s]}^{\infty} \frac{1}{n-1+s}\right) \tag{6}$$

where $S = (\omega/\omega_0) - [\omega/\omega_0]$, $[x]$ denotes the largest integer not exceeding x.

The first two sums in equation (6) may be calculated with the help of the following expansion over a small parameter:

$$\sum_{n=N_1}^{N_2} f(\xi(n-s)) \approx \frac{1}{\xi} \int_{N_1\xi}^{N_2\xi} dx f(x) + (\tfrac{1}{2} - s)(f(N_2\xi) - f(N_1\xi)) + O(\xi). \tag{7}$$

As for the last two sums in equation (6), their calculation, allowing for the smallness of ξ, is readily carried out using simple expansion over s and $1-s$. The final result is

$$\bar{\sigma}_T = \frac{Z^2 e^5 \omega_0^2 E^2 \sin^2\theta}{\hbar \omega^4 m^2 v^3 E_0}(2s-1) \ln\left(\frac{v_E}{v}\right). \tag{8}$$

The dependence $\bar{\sigma}_T(\omega)$ is described in this case by a 'saw'-similar curve. A negative absorption is possible again in the region $\frac{1}{2} < s < 1$, i.e. if $(n-\frac{1}{2})\omega_0 < \omega < n\omega_0$, $n = 1, 2, \ldots$. The cross section $\bar{\sigma}_T$ changes its sign abruptly at the points $\omega = n\omega_0$. The transition region width may be estimated as in the previous case to be $\Delta\omega \approx \omega_0(v/v_E) \ll \omega_0$.

Thus, in both cases considered (equations (4) and (8)) the weak-wave absorption coefficient α decreases asymptotically as E_0^{-1} when $(v_E/v) \to \infty$, in contrast to an absorption of the strong wave itself ($\alpha_0 \sim E_0^{-3}$). This result cannot be obtained by a simple substitution $v \to v_E$ in the corresponding field-free formulae and is probably connected with the difference between uniform and oscillatory motions.

The other important difference from the usual stimulated bremsstrahlung (Marcuse 1962, Bunkin *et al* 1972) is the possibility of amplification in the case of an isotropic electron distribution if $v_E \gg v$. Because of this a comparatively large value of the weak-wave amplification coefficient $\tilde{\alpha}$ may be obtained in principle if we use a plasma with large ion and electron number densities. Using equation (8) we can write, in this case,

$$\tilde{\alpha} = \frac{r_0}{\lambda_0 \lambda} Z \frac{c^3}{v^2 v_E}\left(\frac{\omega_p}{\omega}\right)^4 2\pi \sin^2\theta (2s-1) \ln\left(\frac{v_E}{v}\right) \tag{9}$$

where $r_0 = e^2/mc^2$ is the classical electron dimension, ω_p is the plasma frequency, $\lambda_0 = 2\pi c/\omega_0$, $\lambda = 2\pi c/\omega$. If we take, for example, $Z = 1$, $\theta = \pi/2$, $\omega \approx 3\omega_p$, $\omega \approx \omega_0 \approx 3\times 10^{15}\ \mathrm{s}^{-1}$, $v = 3\times 10^8\ \mathrm{cm\ s}^{-1}$, $v_E = 3\times 10^9\ \mathrm{cm\ s}^{-1}$, with the help of equation (9) we estimate that $\tilde{\alpha} \approx 1\ \mathrm{cm}^{-1}$.

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